II. Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering and Physics, Agricultural and Mechanical College, College Station, Texas.

If the point of projection be origin and the path be regarded as a parabola its equation will be

$$y=x\tan\alpha-\frac{gx^2}{2v^2\cos^2\alpha}\ldots(1),$$

where α=angle of elevation.

At the point where the shell is to strike the plane, $y=-3750$ feet and $x=6495$ feet. Inserting these values in equation (1) there results

$$-3750\times400\cos^2\alpha=6495\times200\times\sin\alpha\cos\alpha-(1299)^2.$$

For $\cos^2\alpha$ write $1+\cos2\alpha$, and for $2\sin\alpha\cos\alpha$ write $\sin2\alpha$, and as $\sin2\alpha=\sqrt{(1-\cos^2 2\alpha)}$, we get

$$\sqrt{(1-\cos^2 2\alpha)}=.722-.577\cos2\alpha\ldots(2).$$

Solving (2) we get $\cos2\alpha=314\pm.676$.

$\therefore$ $2\alpha=8^\circ\ 48'$ or $111^\circ\ 12'$ and $\alpha=4^\circ\ 24'$ or $55^\circ\ 36'$, which values satisfy the condition

$$\alpha''-\tfrac{1}{2}(\tfrac{1}{2}\pi-30)=\tfrac{1}{2}(\tfrac{1}{2}\pi-30)-\alpha'.$$

See Tait and Steele's *Dynamics of a Particle*, page 90.

Also solved by *P. H. PHILBRICK.*

78. Proposed by ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Iowa.

A cone and a cylinder having equal heights and equal circular bases are filled with water; if they have equal holes in the bases, respectively, how many times as long will it take the cylinder to empty as the cone?

I. Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

Let r=radius of base, h=altitude, k=area of orifice, and x=height of the water at the time t.

I. For the cylinder, the discharge in time dt is, $k\sqrt{(2gx)}dt$; and since in the same time the surface of the water descends a distance dx, the quantity in the vessel is *lessened* $\pi r^2 dx$.

$\therefore$ $k\sqrt{(2gx)}dt=\pi r^2 dx$, and

$$t=-\frac{\pi r^2}{k\sqrt{(2g)}}\int\frac{dx}{\sqrt{x}}=-\frac{2\pi r^2}{k\sqrt{(2g)}}x^{\frac{1}{2}}+c=\frac{2\pi r^2}{k\sqrt{(2g)}}(h^{\frac{1}{2}}-x^{\frac{1}{2}}).$$

II. For the cone, we have $y=(r/h)(h-x)$, and the area of section

$$=\pi y^2=\frac{\pi r^2}{h^2}(h-x)^2.$$

$$\therefore\ k\sqrt{(2gx)}dt=\frac{\pi r^2}{h^2}(h-x)^2dx, \text{ and } t=-\frac{\pi r^2}{h^2k\sqrt{(2g)}}\int\frac{(h-x)^2}{x^{\frac{1}{2}}}dx$$

$$=-\frac{\pi r^2}{h^2k\sqrt{(2g)}}\int\frac{(h^2-2hx+x^2)dx}{x^{\frac{1}{2}}}$$

$$=-\frac{\pi r^2}{h^2k\sqrt{(2g)}}\cdot(2h^2x^{\frac{1}{2}}-\frac{4h}{3}x^{\frac{3}{2}}+\tfrac{2}{5}x^{\frac{5}{2}})+c.$$

But $t=0$ for $x=h$. $\therefore c=\dfrac{\pi r^2}{h^2k\sqrt{(2g)}}\,\tfrac{16}{15}h^{\frac{5}{2}}$.

Hence $t=-\dfrac{\pi r^2}{h^2k\sqrt{(2g)}}(2h^2x^{\frac{1}{2}}-\dfrac{4h}{3}x^{\frac{3}{2}}+\tfrac{2}{5}x^{\frac{5}{2}})+\dfrac{\pi r^2}{k\sqrt{(2g)}}\,\tfrac{16}{15}h^{\frac{1}{2}}$.

For $x=0$, the time of emptying vessel$=t_2=\dfrac{\pi r^2}{k\sqrt{(2g)}}.\tfrac{16}{15}h^{\frac{1}{2}}$.

This is $\frac{8}{15}$ of the time of emptying the cylinder.

II. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Let A=the area of the descending surface, O=the area of the orifice, and x=the depth of the water at the end of any time t.

The quantity of water discharged through the orifice in the infinitely small time dt is $O.dt(2gx)^{\frac{1}{2}}$, the velocity of discharge being $(2gx)^{\frac{1}{2}}$; but in the same time the surface has descended through the distance dx and the quantity discharged is Adx.

$\therefore\ O.dt(2gx)^{\frac{1}{2}}=Adx$, or $dt=\dfrac{Adx}{O.(2gx)^{\frac{1}{2}}}$, as general formula for any shape of vessel.

Now for the cylinder $A=\pi r^2$ and therefore $dt=-\dfrac{\pi r^2dx}{O(2gx)^{\frac{1}{2}}}$ and

$$t=-\frac{\pi r^2}{O(2g)^{\frac{1}{2}}}\int_h^o\frac{dx}{x^{\frac{1}{2}}}=\frac{2\pi r^2h^{\frac{1}{2}}}{O(2g)^{\frac{1}{2}}}=\frac{\pi r^2(2h)^{\frac{1}{2}}}{O.g^{\frac{1}{2}}};$$

and for the cone $A=\dfrac{\pi r^2(h-x)^2}{h^2}$ and

$$\therefore\ dt=-\frac{\pi r^2(h-x)^2dx}{Oh^2(2g)^{\frac{1}{2}}} \text{ and } t=-\frac{\pi r^2}{Oh^2(2g)^{\frac{1}{2}}}\int_h^o\frac{(h-x)^2dx}{x^{\frac{1}{2}}}$$

$$\frac{\pi r^2}{Oh^2(2g)^{\frac{1}{2}}}.\frac{16h^{\frac{5}{2}}}{15}=\tfrac{8}{15}.\frac{\pi r^2(2h)^{\frac{1}{2}}}{O.g^{\frac{1}{2}}}.$$

Comparing the two results we see that the cone empties in $\frac{8}{15}$ of the time it takes the cylinder, or the cylinder takes $1\frac{7}{8}$ as long as the cone to empty. The minus sign is prefixed because x decreases as t increases.

Also solved by *G. B. M. ZERR*, *ELMER SCHUYLER*, *J. SCHEFFER*, and *J. C. NAGLE*.

NOTE. In reference to problem 63, Dr. Arnold Emch says: "I had the problem solved by my class in graphic statics by a purely graphical method and the following values (approximations) were obtained: $\angle ABE=46°$, $\angle BAD=56°$, tension in $BE=56.8$, tension in $AD=71$. This shows that the solution in the MONTHLY is correct."

DIOPHANTINE ANALYSIS.

71. Proposed by A. H. BELL, Hillsboro, Ill.

Find five numbers such that the product of any two plus 1 will equal a square.

III. Solution by M. A. GRUBER, A. M., War Department, Washington. D. C.

By using $(s-1)^2$, the denominator of Euler's fifth number, where $s=4n(n-1)(n+1)[4n(2n-1)(2n+1)]$, I have found five numbers in terms of n: $x=n-1$, $y=n+1$, $z=4n$, $w=4n(2n-1)(2n+1)$, and

$$v=\frac{4n(2n-1)(2n+1)[2n(2n-1)-1][2n(2n+1)-1](8n^2-1)}{\{4n(n-1)(n+1)[4n(2n-1)(2n+1)]-1\}^2}.$$

The numerator of v is four times the product of the roots of the six squares $xy+1$, $xz+1$, $yz+1$, $xw+1$, $yw+1$, and $zw+1$.

The denominator of v is the square of $(xyzw-1)$.

Take $n=1, 2, 3, 4, 5, 6$, etc. We then obtain the following sets of five numbers:

$0, \quad 2, \quad 4, \quad 12, \quad 420;$

$1, \quad 3, \quad 8, \quad 120, \quad \frac{777480}{(2879)^2};$

$2, \quad 4, \quad 12, \quad 420, \quad \frac{35455980}{(40319)^2};$

$3, \quad 5, \quad 16, \quad 1008, \quad \frac{499902480}{(241919)^2};$

$4, \quad 6, \quad 20, \quad 1980, \quad \frac{3822388020}{(950399)^2};$

$5, \quad 7, \quad 24, \quad 3432, \quad \frac{20000100120}{(2882879)^2};$ etc., etc., etc.

We also find